

June 27, 2013

Calculators not allowed. **Show your work for any possible partial credit or in some cases for any credit at all.** Note the last page shows you some blank K maps just in case you need them on this exam. 8 pages, possible 100 points.

Problem 1 Part A (10 points) Convert the following notations:

Binary representation	Hexidecimal representation
1010110010010101	A C 9 5
Signed Binary two's complement represented	Decimal
11100.01	- 3.75
Decimal	Octal
673	1241
Decimal Notation	Binary notation
7.75	111.11

$$111.00.01 \Rightarrow 00011.10 + 1 \Rightarrow 00011.11 \Rightarrow 3 + \frac{1}{2} + \frac{1}{2^2} = 3.75$$

673

$$\begin{array}{r}
 336 \\
 2 \overline{)673} \quad R1 \\
 168 \\
 \hline
 336 \\
 2 \overline{)336} \quad R0 \\
 84 \\
 \hline
 168 \\
 2 \overline{)168} \quad R0 \\
 42 \\
 \hline
 84 \\
 2 \overline{)84} \quad R0 \\
 21 \\
 \hline
 42 \\
 2 \overline{)42} \quad R0 \\
 21 \\
 \hline
 10 \\
 2 \overline{)21} \quad R1 \\
 5 \\
 \hline
 10 \\
 2 \overline{)10} \quad R0 \\
 2 \\
 \hline
 5 \\
 2 \overline{)5} \quad R1 \\
 1 \\
 \hline
 2 \\
 2 \overline{)2} \quad R0 \\
 0 \\
 \hline
 1 \\
 2 \overline{)1} \quad R1
 \end{array}
 \rightarrow 1\circled{0}1\circled{0}1\circled{0}0\circled{0}1$$

$$7.75 \Rightarrow 111.11$$

$$\begin{array}{r}
 | \\
 1 \times 2^{-1} \\
 \swarrow \\
 1 \times 2^{-2}
 \end{array}$$

Problem 1 Part B (15 points) For the 24 (and 20) bit representations below, determine the most negative value, most positive value, and step size (difference between sequential values). All answers must be expressed in decimal notation. Fractions (for example 3/16ths) may be used. All signed representations are two's complement signed numbers.

representation	most negative value	most positive value	step size
unsigned integer (24 bits). (0 bits)	0	$16M - 1$	1
Signed integer (24 bits). (0 bits)	$-8M$	$8M - 1$	1
unsigned fixed-point (17 bits). (7 bits)	0	$128K - \frac{1}{128}$	$\frac{1}{128}$
signed fixed-point (15 bits). (5 bits)	$-16K$	$16K - \frac{1}{32}$	$\frac{1}{32}$

$$\text{UNSIGNED} : \frac{2^N - 1}{2^k}$$

$$\frac{2^{24} - 1}{2^0} = 2^{24} - 1 = 2^{10}2^{10}2^4 - 1 = 16M - 1$$

$$\text{SIGNED} : -\frac{2^{N-1}}{2^k} +_0 \frac{2^{N-1}}{2^k} \Rightarrow -\frac{2^{23}}{2^0} +_0 \frac{2^{23}}{2^0} \Rightarrow -2^{10}2^{10}2^3 +_0 2^{10}2^{10}2^3 = -8M +_0 8M - 1$$

UNSIGNED fixed point

$$\frac{2^{24} - 1}{2^7} \Rightarrow 2^{17} - \frac{1}{2^7} \Rightarrow 2^{10}2^7 - \frac{1}{2^7} \Rightarrow 128K - \frac{1}{128}$$

SIGNED

$$-\frac{2^{19}}{2^5} +_0 \frac{2^{19}}{2^5} \Rightarrow -2^{14} +_0 2^{14} - \frac{1}{2^5} \Rightarrow -2^{10}2^4 +_0 2^{10}2^4 - \frac{1}{2^5} \\ \Rightarrow -16K +_0 16K - \frac{1}{32}$$

Problem 2 Part A Arithmetic (5 points) For each problem below, compute the operations using the rules of arithmetic, and indicate whether an overflow occurs assuming all numbers are expressed using a six bit **unsigned** representation

$$\begin{array}{r}
 \begin{array}{r}
 \begin{array}{r}
 110110 \\
 +100110
 \end{array} \\
 \hline
 101100
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{r}
 011001 \\
 +010110
 \end{array} \\
 \hline
 101111
 \end{array}
 \end{array}$$


---

result      011100      101111

---

unsigned  
error?      Y      N

---

Problem 2 Part B Arithmetic (10 points) For each problem below, compute the operations using the rules of arithmetic, and indicate whether an overflow occurs assuming all numbers are expressed using a six bit **signed** two's complement representations.

$$\begin{array}{r}
 \begin{array}{r}
 \begin{array}{r}
 011111 \\
 +000100
 \end{array} \\
 \hline
 100011
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{r}
 111111 \\
 +000010
 \end{array} \\
 \hline
 100000\ 01
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{r}
 011000 \\
 -111011
 \end{array} \\
 \hline
 011101
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{r}
 001111 \\
 -010101
 \end{array} \\
 \hline
 111010
 \end{array}
 \end{array}$$


---

result      100011      000001      011101      111010

---

signed  
error?      Y      N      N      N

---

$$\begin{array}{r}
 \begin{array}{r}
 \begin{array}{r}
 111011 \\
 000100 \\
 +1
 \end{array} \\
 \hline
 000101
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{r}
 010101 \\
 101010 \\
 +1
 \end{array} \\
 \hline
 101011
 \end{array}
 \end{array}$$
  

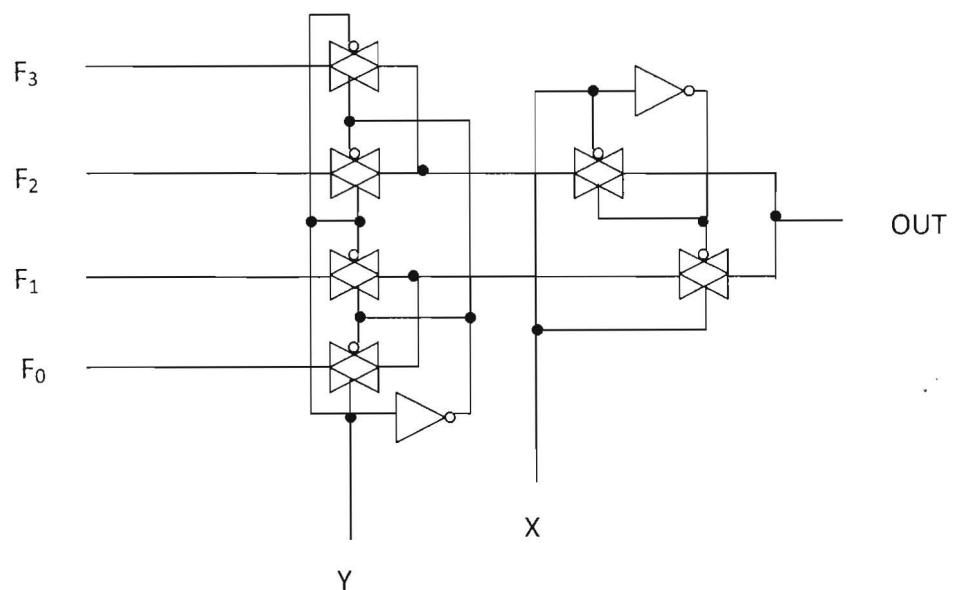
$$\begin{array}{r}
 \begin{array}{r}
 \begin{array}{r}
 011000 \\
 000101 \\
 +1
 \end{array} \\
 \hline
 011101
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{r}
 001111 \\
 101011 \\
 +1
 \end{array} \\
 \hline
 111010
 \end{array}
 \end{array}$$

3) Given the circuit below:

- a) (5 points) The designer wants to implement an exclusive-or function where  $\text{out} = Y$  exclusive-or  $X$ . Fill in the values for  $F_3 F_2 F_1 F_0$  to implement the desired exclusive-or function.
- b) (5 points) The designer wants to implement the NAND function where  $\text{out} = Y \text{ NAND } X$ . Fill in the values for  $F_3 F_2 F_1 F_0$  to implement the desired NAND function.
- c) (5 points) The designer wants to implement the NOR function where  $\text{out} = Y \text{ NOR } X$ . Fill in the values for  $F_3 F_2 F_1 F_0$  to implement the desired NOR function.

function desired:	$F_3$	$F_2$	$F_1$	$F_0$
a) exclusive-or	0	1	1	0
b) NAND	1	1	1	0
c) NOR	1	0	0	0

$Y \quad X$		Ex-or	NAND	NOR
0 0	$F_3$	0	1	1
0 1	$F_1$	1	1	0
1 0	$F_2$	1	1	0
1 1	$F_0$	0	0	0

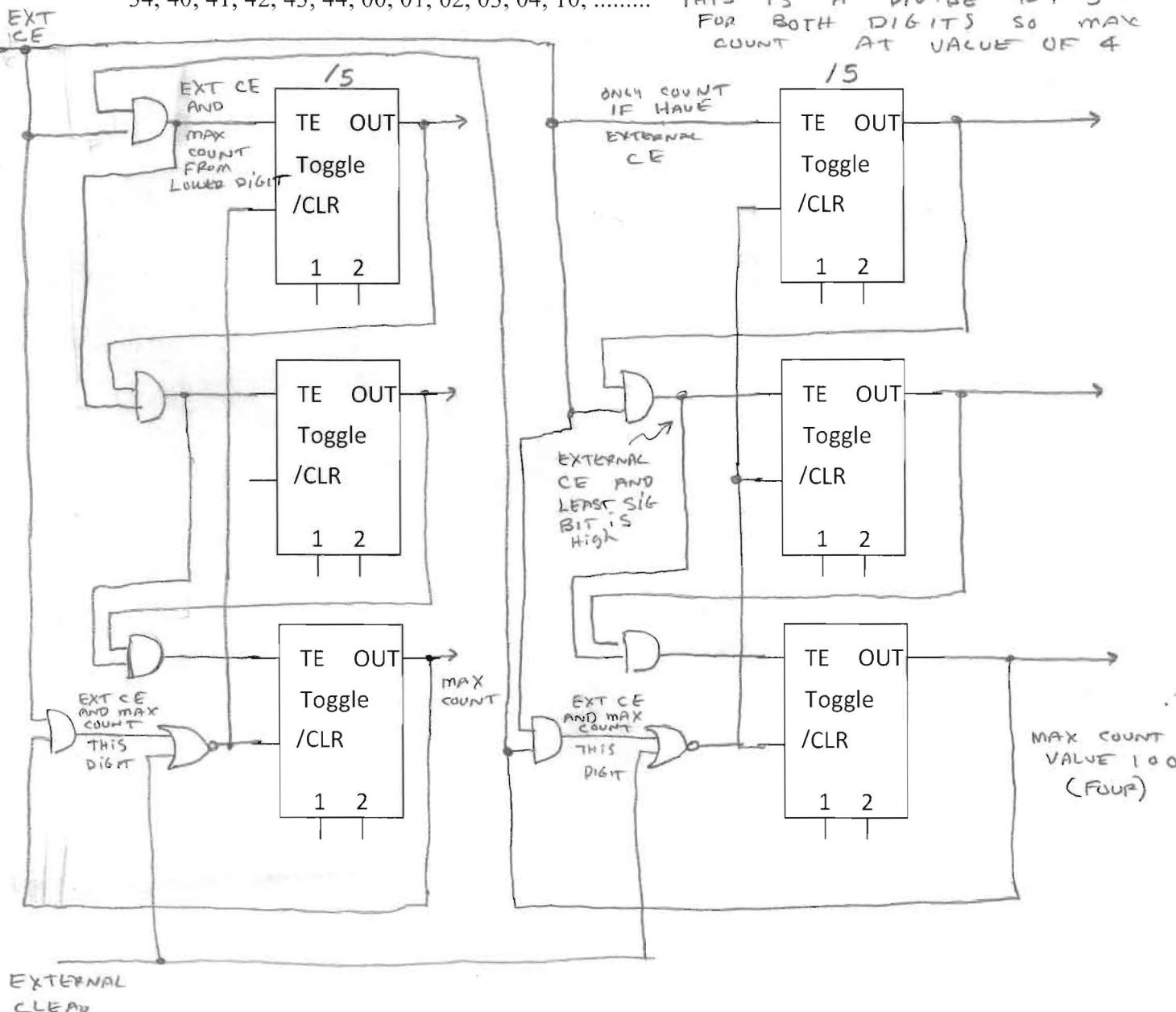


- 4) Counters (15 points) Connect the needed toggle cells below to build a multiple digit counter (like in an alarm clock type application) that counts in the following strange 2 digit sequence. Include any circuitry needed to allow this to work. The toggle cells in the right column are for the least significant digit (right value), the toggle cells in the left column are for the most significant digit (left value). Include an active high count enable (CE) and an active high reset (RESET) inputs.

COUNT SEQUENCE
000
001
010
011
100

desired count sequence: 00, 01, 02, 03, 04, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 00, 01, 02, 03, 04, 10, .......

THIS IS A DIVIDE BY 5  
FOR BOTH DIGITS SO MAX COUNT AT VALUE OF 4



NOTE: ALL CLK 1'S CONNECTED  
ALL CLK 2'S CONNECTED

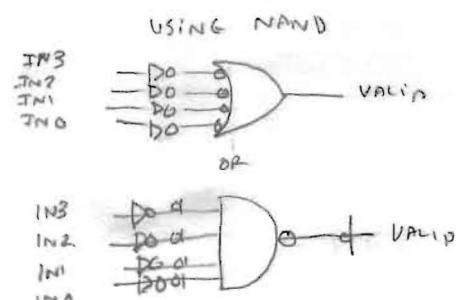
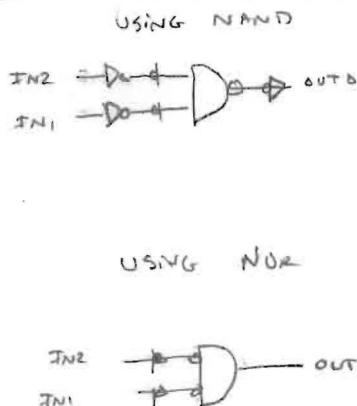
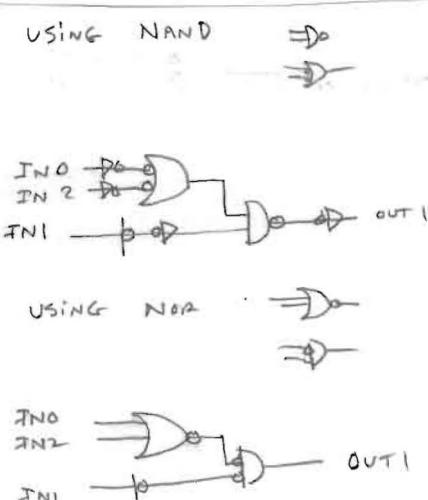
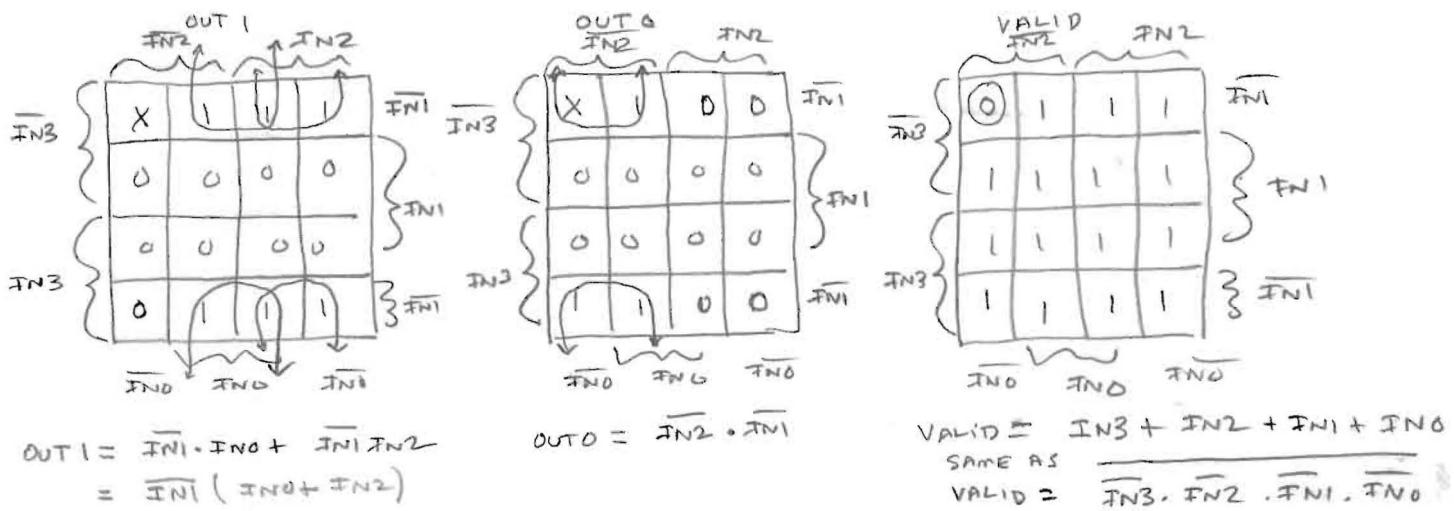
5) Priority Encoders (15 points) Given the truth table for the following priority encoder:

IN3	IN2	IN1	IN0	OUT1	OUT0	VALID
0	0	0	0	X	X	0
1	0	0	0	0	1	1
X	1	0	X	1	0	1
X	X	1	X	0	0	1
X	0	0	1	1	1	1

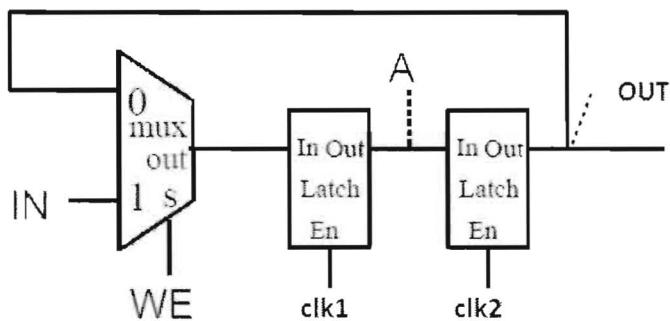
a) List the priority order of the inputs IN3, IN2, IN1, IN0:

$$\frac{\text{IN1}}{\text{highest}} > \frac{\text{IN2}}{\text{ }} > \frac{\text{IN0}}{\text{ }} > \frac{\text{IN3}}{\text{lowest}}$$

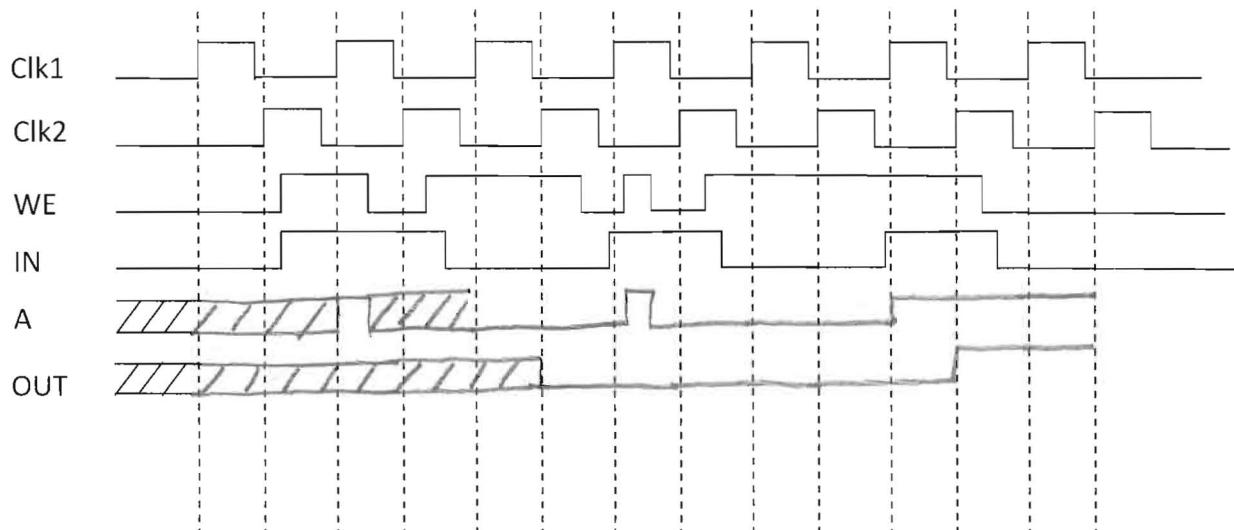
b) Using basic gates (AND, OR, NAND, NOR, NOT) show the gate level implementation for this priority encoder.



6) Registers (15 points) Consider the register implementation below.



Assume the following signals are applied to your register. Draw the signal at point A (output of the first latch), the signal at point OUT (output of second latch). Assume A and OUT start at unknown values.



K MAPS:

